Engineering Economics Knowledge

- 4 – 6 problems
- Discounted cash flow
  - Equivalence, PW, equivalent annual worth, FW, rate of return
- Cost
  - Incremental, average, sunk, estimating
- Analyses
  - Breakeven, benefit-cost, life cycle
- Uncertainty
  - Expected value and risk
- Sections in FE Reference Handbook
  - Engineering Economics

Engineering Economics

- Year-End & Other Conventions
- Cash Flow
- Discount Factors
- Single Payment Equivalence
- Depreciation
- Nonannual Compounding
- Uniform Gradient Equivalence
- Book Value
- Capitalized Cost
- Inflation
- Uniform Series Equivalence
- Break-Even Analysis
- Benefit-Cost Analysis
Year-End & Other Conventions

- Year-end convention: Assume that all receipts and disbursements (cash flows) take place at the end of the year in which they occur.
  - Exceptions:
    - Initial project cost (purchase cost)
    - Trade-in allowance
    - Other cash flows that are associated with the inception of the project at \( t = 0 \)

Cash Flow

- Cash flows = the sums of money recorded as receipts or disbursements in a project’s financial records.
- Cash flow diagrams
  - Can be drawn to help visualize and simplify problems that have diverse receipts and disbursements
  - Horizontal (time) axis is marked off in equal increments, one per period
  - Receipts
  - Disbursements
  - 2 or more transfers in same period are placed end to end or combined
  - Sunk costs = expenses incurred before \( t = 0 \)

Cash Flow Continued

- Uniform series cash flow
  - Series of equal transactions starting at \( t = 1 \) and ending at \( t = n \)
  - Symbol \( A \) (annual amount) given to the magnitude of each individual cash flow
- Gradient series cash flow
  - Starts with a cash flow (symbol \( G \)) at \( t = 2 \) and increases by \( G \) each year until \( t = n \), at which time the final cash flow is \( (n - 1)G \).
  - Gradient at \( t = 1 \) is 0
Discount Factors

- The procedure for determining the equivalent amount is known as discounting
- Discount factors - symbolic and formula form

### Values of cash flow (discounting) factors are in tabulation form in Handbook
- Pages 133-137

### Single Payment Equivalence

**Single Payment Future Worth**

- \((1 + \frac{1}{n})^n\)
  - Single payment compound amount factor
- Common convention to substitute the standard functional notation of \((F / P, i\%, n)\)
- “Find \(F\), given \(P\), using an interest rate of \(i\%\) over \(n\) years”
Single Payment Equivalence

Single Payment Future Worth Example

A 40-year-old commuting engineer wants to set up a retirement fund to be used starting at age 65. $50,000 is invested now at 6% compounded annually. The amount of money that will be in the fund at retirement is most nearly

(A) $844,000
(B) $690,000
(C) $892,000
(D) $920,000

Single Payment Equivalence

Single Payment Present Worth

<table>
<thead>
<tr>
<th>Single Payment Present Worth</th>
<th>( P )</th>
<th>( (P/P, i, n) )</th>
<th>( (1 + i)^{-n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1 + i)^{-n} )</td>
<td></td>
<td></td>
<td>Single payment present worth factor</td>
</tr>
</tbody>
</table>

Single Payment Equivalence

Single Payment Present Worth Example

$2000 will become available on January 1 in year 8. If interest is 8%, what is most nearly the present worth of this sum on January 1 in year 1?

(A) $1180
(B) $1200
(C) $120
(D) $1220
Uniform Series Equivalence

**Uniform Series Future Worth**

<table>
<thead>
<tr>
<th>Uniform Series Future Worth</th>
<th>( F = A \times (F/A, i, n) )</th>
<th>( F = A \times (F/A, i, n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>• ( A ) = annual amount = cash flow that repeats at the end of each year for ( n ) years without change in amount</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Uniform series compound factor convert from an annual amount to future amount</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Uniform Series Equivalence

**Uniform Series Future Worth Example**

$20,000 is deposited at the end of each year into a fund earning 5% interest. At the end of ten years, the amount accumulated is most nearly:

(A) $150,000
(B) $210,000
(C) $260,000
(D) $280,000

Uniform Series Equivalence

**Uniform Series Annual Value of a Sinking Fund**

<table>
<thead>
<tr>
<th>Uniform Series Sinking Fund</th>
<th>( t ) from ( F )</th>
<th>( (A/F, i, n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Sinking fund = a fund or account into which annual deposits of ( A ) are made in order to accumulate ( F ) at ( t = n ) in the future</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Sinking fund factor = ( (A/F) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Uniform Series Equivalence
Uniform Series Annual Value of a Sinking Fund Example

At the end of each year, an investor deposits some money into a fund earning 7% interest. The same amount is deposited each year, and after six years the account contains $1000. The amount deposited each time is most nearly

(A) $100
(B) $200
(C) $300
(D) $500

Uniform Series Equivalence
Uniform Series Present Worth

- Annuity = a series of equal payments, A, made over a period of time

Uniform Series Equivalence
Uniform Series Present Worth Example

A sum of money is deposited into a fund at 7% interest. $600 is withdraw at the end of each year for nine years, leaving nothing in the fund at the end. The amount originally deposited is most nearly

(A) $1800
(B) $1900
(C) $2000
(D) $3100
Uniform Series Equivalence

Uniform Series Annual Value Using the Capital Recovery Factor

- Capital recovery factor = used when comparing alternatives with different lifespans
- Can convert the present value of each alternative into its equivalent annual value, using the assumption that each alternative will be renewed repeatedly up to the duration of the longest-lived alternative

Uniform Gradient Equivalence

Uniform Gradient Present Worth

- Used to find present worth, P, of a cash flow that is increasing by a uniform amount, G.

Uniform Gradient Equivalence

Uniform Gradient Future Worth

- Used to find future worth, F, of a cash flow that is increasing by a uniform amount, G.
Uniform Gradient Equivalence
Uniform Gradient Uniform Series Factor

<table>
<thead>
<tr>
<th>Uniform Gradient Uniform Series Factor</th>
<th>( G )</th>
<th>( G )</th>
<th>( \frac{1 - \frac{n}{i}}{\frac{n}{i} - 1} )</th>
</tr>
</thead>
</table>

- Used to find the equivalent annual worth, \( A \), of a cash flow that is increasing by a uniform amount, \( G \).

Non-Annual Compounding
Effective Annual Interest Rate

- If there are \( m \) compounding periods during the year (2 for semiannual compounding, 4 for quarterly compounding, 12 for monthly compounding, etc.), the effective interest rate per period, \( i_c \), is \( i / m \).

\[ i_c = \left( 1 + \frac{i}{m} \right)^{\frac{m}{i}} - 1 \]

- Effective annual interest rate, \( i_c \), can be calculated from the effective interest rate per period.
Non-Annual Compounding

Effective Annual Interest Rate Example

Money is invested at 2% per annum and compounded quarterly. The effective annual interest rate is most nearly

(A) 5.1%
(B) 5.2%
(C) 3.4%
(D) 5.5%

Depreciation

Straight Line Method

- Depreciation is the same each year
- Depreciation basis is divided uniformly among all of the years in the depreciation periods
- With the straight line method, depreciation is the same each year
  - Depreciation basis (\(C - S_n\)) is divided uniformly among all of the \(n\) years in the depreciation period.

\[
D_n = \frac{C - S_n}{n}
\]

\[\text{p. 132}\]

Depreciation

Straight Line Method Example

A computer will be purchased at $1800. The expected salvage value at the end of its service life of 10 years is $100. Using the straight line method, the annual depreciation for this computer is most nearly

(A) $210
(B) $250
(C) $290
(D) $290
Depreciation

Modified Accelerated Cost Recovery System (MACRS)

• Cost recovery amount in a particular year is calculated by multiplying the initial cost of the asset by a factor
  – Initial cost is not reduced by the asset’s salvage value
• Factor to be used varies depending on the year and on the total number of years in the asset’s cost recovery period
  – Subject to continuing legislation changes

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.33</td>
<td>20.00</td>
<td>13.90</td>
<td>10.00</td>
</tr>
<tr>
<td>2</td>
<td>18.44</td>
<td>12.00</td>
<td>8.64</td>
<td>6.67</td>
</tr>
<tr>
<td>3</td>
<td>11.84</td>
<td>11.84</td>
<td>7.49</td>
<td>5.63</td>
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<tr>
<td>4</td>
<td>7.94</td>
<td>10.41</td>
<td>6.49</td>
<td>5.00</td>
</tr>
<tr>
<td>5</td>
<td>5.76</td>
<td>6.62</td>
<td>4.32</td>
<td>3.33</td>
</tr>
<tr>
<td>6</td>
<td>4.44</td>
<td>5.00</td>
<td>3.20</td>
<td>2.50</td>
</tr>
<tr>
<td>7</td>
<td>3.64</td>
<td>4.25</td>
<td>2.63</td>
<td>2.03</td>
</tr>
<tr>
<td>8</td>
<td>3.03</td>
<td>3.64</td>
<td>2.22</td>
<td>1.72</td>
</tr>
<tr>
<td>9</td>
<td>2.52</td>
<td>3.00</td>
<td>1.81</td>
<td>1.44</td>
</tr>
<tr>
<td>10</td>
<td>2.11</td>
<td>2.52</td>
<td>1.61</td>
<td>1.29</td>
</tr>
</tbody>
</table>

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Depreciation

Modified Accelerated Cost Recovery System (MACRS) Example

A groundwater treatment system costs $1,500,000. It is expected to operate for a total of 120,000 hours over a period of 10 years, and then have a $250,000 salvage value. During the system’s first year in service, it is operated for 6000 hours. Using the MACRS method, its depreciation in the third year is most nearly:

(A) $160,000
(B) $250,000
(C) $360,000
(D) $400,000

Book Value

• Difference between the original purchase price and the accumulated depreciation is known as the book value, BV
• Book value is initially equal to the purchase price, and at the end of each year it is reduced by that year’s depreciation

BOOK VALUE

BV = initial cost – ΣD_i

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Book Value Example

A machine initially costing $25,000 will have a salvage value of $6000 after five years. Using MACRS depreciation, its book value after the third year will be most nearly:

(A) $5000  
(B) $7200  
(C) $3000  
(D) $14,600

Capitalized Cost

- Present worth of a project with an infinite life is known as the capitalized cost.
- Capitalized cost is the amount of money at \( t=0 \) needed to perpetually support the project on the earned interest only.
- Capitalized cost is a positive number when expenses exceed income.
- Equation can be used when the annual costs are equal in every year.

Capitalized Cost Example

The construction of a volleyball court will cost $1200, and annual maintenance cost is expected to be $300. As an effective annual interest rate of 5%, the project’s capitalized cost is most nearly:

(A) $1500  
(B) $1900  
(C) $1700  
(D) $20,000
Inflation

• \( f \) is a constant inflation rate per year
• Inflation-adjusted interest rate, \( d \), can be used to compute present worth.

Inflation Example

An investment of $20,000 earns an effective annual interest of 20%. The value of the investment in five years, adjusted for an annual inflation rate of 6%, is closest to

(A) $27,000
(B) $25,000
(C) $24,000
(D) $24,000

Break-Even Analysis

• Method of determining when the value of one alternative becomes equal to the value of another.
Benefit-Cost Analysis

- Used in municipal project evaluations where benefits and costs accrue to different segments of the community
- Present worth of all benefits, $B$, is divided by the present worth of all costs, $C$
- If the benefit-cost ratio, $B/C$, is greater than or equal to 1, the project is acceptable
  - Equivalent uniform annual costs can be used in place of present worths

Benefit-Cost Analysis Example

A large sewer system will cost $170,000 annually. There will be favorable consequences to the potential public equivalency to $100,000 annually, and adverse consequences to a small segment of the public equivalent to $50,000 annually. The benefit-cost ratio is most nearly

(A) 2.2
(B) 2.4
(C) 2.6
(D) 2.8