Fundamentals of Engineering REVIEW COURSE For Civil Engineers

Fluid Mechanics
Fluid Mechanics

- Fluid Properties
- Fluid Statics
- Fluid Flow Measurement
- Bernoulli's Equation
- Venturi Meter
- Pitot Tube
Fluid Properties

What is the weight of 3.0 slugs when the acceleration due to gravity is 31.7 ft./sec²

\[ F = W = (3.0)(31.7) = 95.1 \text{ lb.} \]

If 12 m³ of oil weighs 100000 N., calculate its specific weight, density, specific gravity.

\[ \gamma = \frac{W}{V} = \frac{100000}{12} = 8333 \text{ N/ m}^3 \]

\[ \rho = \frac{\gamma}{g} = \frac{8333}{9.81} = 850 \text{ Kg/m}^3 \]

\[ \text{s.g.} = \frac{\gamma_{\text{oil}}}{\gamma_{\text{H}_2\text{O}}} = \frac{8333}{9810} = .85 \]

If 300 ft³ of oil weighs 10,520 lb., calculate its specific weight, density, specific gravity.

\[ \gamma = \frac{W}{V} = \frac{10,520}{300} = 52.6 \text{ lb./ ft}^3 \]

\[ \rho = \frac{\gamma}{g} = \frac{52.6}{32.2} = 1.63 \text{ slugs/ft}^3 \]

\[ \text{s.g.} = \frac{\gamma_{\text{oil}}}{\gamma_{\text{H}_2\text{O}}} = \frac{52.6}{62.4} = .843 \]
Fluid Properties

What are the components of absolute pressure for a positive pressure system?

(A) atmospheric and barometric

(B) atmospheric and differential

(C) barometric and gage

(D) vacuum and gage

Because atmospheric pressure is taken at the time the gage pressure is measured, the atmospheric pressure equals barometric pressure. Absolute pressure is the sum of gage pressure and barometric pressure for positive pressures and is the difference between barometric pressure and vacuum for negative pressures.
Fluid Properties

On a day when the barometric pressure is 760 mm Hg, a vacuum gauge indicates 500 mm Hg. What is the approximate absolute pressure corresponding to the gauge reading? \( \rho_{\text{Hg}} = 13600 \text{ kg/m}^3 \)

\[
\begin{align*}
\text{(A)} & \quad 3.5 \text{ kPa} \\
\text{(B)} & \quad 13 \text{ kPa} \\
\text{(C)} & \quad 35 \text{ kPa} \quad \text{Correct Answer} \\
\text{(D)} & \quad 47 \text{ kPa}
\end{align*}
\]

\[
\begin{align*}
\rho_{\text{abs}} &= \rho_{\text{atm}} + \rho_{\text{gage}} = \rho_{\text{atm}} - \rho_{\text{vacuum}} \\
&= (h_{\text{atm}} - h_{\text{vacuum}}) \rho g \\
&= (760 \text{ mm} - 500 \text{ mm}) \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) \\
& \quad \times \left( 13600 \frac{\text{ kg}}{\text{ m}^3} \right) \left( 9.8 \frac{\text{ m}}{\text{s}^2} \right) \\
&= 34653 \text{ Pa} \quad (35 \text{ kPa})
\end{align*}
\]
Fluid Properties

The Reynolds number may be calculated from

(A) diameter, velocity, and absolute viscosity

(B) diameter, velocity, and kinematic viscosity

(C) diameter, density, and absolute viscosity

The ratio of the absolute viscosity to the density, \( \mu/\rho \), is known as the kinematic viscosity, \( \nu \).

\[
\text{Re} = \frac{\rho vD}{\mu} = \frac{\rho}{\mu} vD = \frac{vD}{\nu}
\]
Fluid Properties

How do velocity and viscosity, $\mu$, influence the value of the drag coefficient?

The drag coefficient is related to the Reynolds number by

$$C_D = \frac{24}{Re}$$

Using $D$ as the diameter of flow, the Reynolds number can be defined by

$$Re = \frac{Dv\rho}{\mu}$$

Combining these equations gives

$$C_D = \frac{24\mu}{Dv\rho}$$

This equation shows a direct relationship between $C_D$ and $\mu$ and an inverse relationship between $C_D$ and $v$. Therefore, as $v$ increases, $C_D$ decreases, and as $\mu$ increases, $C_D$ increases.
Fluid Properties

Four cars, each with mass 3200 lbm, are loaded on a 20 ft wide, 40 ft long small-car ferry. Assume the ferry is weightless. How deep will the ferry sink in the water?

The total weight of the loaded vessel must equal the weight of the displaced water.

\[ W_{\text{ferry}} = 4W_{\text{car}} = 4m_{\text{car}}g \]
\[ = \rho_{\text{water}}gV_{\text{ferry}} \]
\[ = \rho_{\text{water}}g(Lwd) \]

\[
d = \frac{4m_{\text{car}}}{\rho_{\text{water}}Lw}
\]
\[
= \frac{(4)(3200 \text{ lbm})}{\left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)(40 \text{ ft})(20 \text{ ft})}
\]
\[
= (0.256 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)
\]
\[
= 3.1 \text{ in} \quad (3 \text{ in})
\]
Fluid Statics

For the configuration shown below calculate the weight of the piston if the gauge pressure reading is 70 kPa.

\[ W = \text{weight of the piston} \]

\[ \frac{W}{\pi (1)^2/4} - (0.86)(9.8)(1) = 70 \text{ kPa} \Rightarrow W = 61.6 \text{ kN} \]
Fluid Statics

A pump that is 85% efficient provides 1.2 m³ of water each minute to a holding tank, as shown in the following figure. The cast iron pipe is 6 cm in diameter. Assume that atmospheric pressure is 100 kPa and that the specific weight of water is 9800 N/m³.

Before the pump is turned on, the pressure at the pump exit is most nearly

When the pump is not operating, the pressure is equal to the hydrostatic pressure at the given location. The head, \( h \), is equal to the difference in elevation between the top of the holding tank and the pump exit.

Multiplying the head at the pump exit by the specific weight of water, \( \gamma \), gives the pressure at the exit of the pump, \( p \).

\[
p = \gamma h = \left( 9800 \frac{N}{m^3} \right) (100 \text{ m} - 75 \text{ m}) = 245000 \text{ Pa} (250 \text{ kPa})
\]

\[
\begin{align*}
(A) & \quad 110 \text{ kPa} \\
(B) & \quad 150 \text{ kPa} \\
(C) & \quad 250 \text{ kPa} \\
(D) & \quad 320 \text{ kPa}
\end{align*}
\]
For the following illustration, the height, $h$, is 25 cm. The specific weight and gravity of water are 9800 N/m$^3$ and 1, respectively. The difference in pressure between the water and the oil is most nearly

Use a pressure balance at the level of the water-mercury interface to find the pressure difference. The pressure from the water must equal the pressure from the oil and the mercury. Include the pressures from the water and oil reservoirs.

\[
\text{\textbf{pressure}}_{\text{water}} = \text{\textbf{pressure}}_{\text{oil}} + \text{\textbf{pressure}}_{\text{mercury}}
\]

\[
p_w + SG_w \gamma h = p_o + SG_o \gamma (2h) + (SG_m \gamma h)
\]

Solve for the pressure difference.

\[
p_w - p_o = \gamma h (2SG_o + SG_m - SG_w)
\]

\[
= \left( 9800 \ \frac{N}{m^3} \right) (0.25 \ \text{m})((2)(0.8) + 13.6 - 1)
\]

\[
= 34,790 \ \text{Pa}(35 \ \text{kPa})
\]
Fluid Statics

A steel tank is filled to a depth of 18 m with a liquid whose specific weight, $\gamma$, is given by the formula $\gamma = \gamma_{H,O}(1 + 0.018y)$, where $y$ is the depth below the surface of the liquid as illustrated. If the radius of the tank is 5.0 m, the tank material has a maximum tensile stress of 240 MPa, and a safety factor of 1.5 is desired, what is the minimum thickness of the tank walls?

Consider a thin layer of liquid situated at an arbitrary distance $y$ from the surface of the liquid.

\[ \sum F_y = (P + dP)A - PA - \gamma Ady = 0 \]
\[ dP = \gamma dy \]

\[ \int dP = \int \gamma_{H,O}(1 + 0.018y)dy \]

\[ P = \gamma_{H,O}(y + \frac{0.018y^2}{2}) + C \]

$C = 0$ since $P = 0$ at surface
Fluid Statics

From Previous Slide

\[ P = \gamma_{H2O}(y + \frac{0.018y^2}{2}) \]

The maximum pressure is at the bottom of the cylinder.

\[ \gamma_{H2O} = 9810 \text{ N/m}^3 \]

\[ P_{\text{max}} = \left(9810 \frac{\text{N}}{\text{m}^3}\right) \left(18 \text{ m} + \frac{(0.018)(18 \text{ m}^2)}{2}\right) \]

\[ = 205186 \text{ N/m}^2 \]

The axial stress is

\[ \sigma_a = \frac{P_r}{2t_a}, \quad t_a = \frac{P_r}{2\sigma_a} \]

The hoop stress is

\[ \sigma_h = \frac{P_r}{t_r}, \quad t_r = \frac{P_r}{\sigma_h} \]

For a constant wall thickness, \( \sigma_h > \sigma_a \)

So \( \sigma_h \) determines the wall thickness.

The design stress is

\[ \sigma_h = \frac{\sigma_{\text{max}}}{SF} = \frac{240}{1.5} = 160 \text{ M Pa} \]

The minimum wall thickness is

\[ t_r = \frac{P_r}{\sigma_h} = \frac{\left(205186 \frac{\text{N}}{\text{m}^2}\right)(5 \text{ m})}{(160 \text{ MPa})\left((10)^6 \frac{\text{Pa}}{\text{MPa}}\right)} \]

\[ = 0.0064 \text{ m} \quad (6.4 \text{ mm}) \]
**Fluid Statics**

A force is applied at the mid-depth of a gate to keep it closed. The force $P$ required per meter of gate width is most nearly

$$F = \gamma h_c A = \rho gh_c bw$$

$$= \left(1000 \ \frac{\text{kg}}{\text{m}^3}\right) \left(9.8 \ \frac{\text{m}}{\text{s}^2}\right) \left(\frac{5 \text{ m}}{2} \left(\sin 60^\circ\right)\right) (5 \text{ m})(1 \text{ m})$$

$$= 106000 \text{ N}$$

Take moments about the hinge.

$$\Sigma M = \left(106,000 \text{ N}\right) \left(\frac{5 \text{ m}}{3}\right) - F \left(\frac{5 \text{ m}}{2}\right) = 0$$

$$F = 70667 \text{ N} \quad (71 \text{ kN})$$

The pressure distribution is triangular. Mathematically,

$$p = 0 \text{ at the surface}$$

$$p = \gamma h \text{ at the hinge}$$

The force of the water on the gate acts through the centroid of the triangular distribution, which is $5/3$ m from the hinge. The force of the water is determined from the average pressure at the average depth. Moments about the hinge give
A 2.0 m long hinged gate is placed in a large water tank such that the hinge, which is located at the top of the gate, is 0.40 m below the free surface of the water as illustrated. If the width of the gate is 0.30 m, most nearly what force, $F$, must be applied at the bottom of the gate to prevent the gate from swinging open?

First find the pressure distribution of the gate.

$$P = \gamma h$$

$h$ is measured from the surface down.

$$\gamma = 9.81 \text{ kN/m}^3 \quad \text{[for water]}$$

Let $P_1 =$ pressure at the hinge and $P_2 =$ pressure at the gate bottom.

$$P_1 = \gamma h_1 = \left(9.81 \text{ kN/m}^3\right)(0.40 \text{ m}) = 3.92 \text{ kPa}$$

$$P_2 = \gamma h_2 = \left(9.81 \text{ kN/m}^3\right)(2.4 \text{ m}) = 23.5 \text{ kPa}$$
Fluid Statics

A 2.0 m long hinged gate is placed in a large water tank such that the hinge, which is located at the top of the gate, is 0.40 m below the free surface of the water as illustrated. If the width of the gate is 0.30 m, most nearly what force, \( F \), must be applied at the bottom of the gate to prevent the gate from swinging open?

The force due to the pressure distribution can be replaced by two equivalent forces, \( F_1 \) and \( F_2 \), acting at the centroid of areas 1 and 2, as illustrated.

\[
F = PA \\
F_1 = \left( 3.92 \ \frac{kN}{m^2} \right)(2.0 \ m)(0.3 \ m) \\
\quad = 2.35 \ kN \quad \text{[acting 1 m below the hinge]} \\
F_2 = \left( 23.5 \ \frac{kN}{m^2} - 3.92 \ \frac{kN}{m^2} \right) \left( \frac{1}{2} \right)(2.0 \ m)(0.3 \ m) \\
\quad = 5.88 \ kN \quad \text{[acting 1.33 m below the hinge]}
\]

To find the force required to keep the gate shut, sum the moments about the hinge.

\[
\sum M = F_1 L_1 + F_2 L_2 - F(2.0 \ m) = 0 \\
F = \frac{F_1 L_1 + F_2 L_2}{2.0 \ m} \\
\quad = \frac{(2.35 \ kN)(1.0 \ m) + (5.88 \ kN)(1.33 \ m)}{2.0 \ m} \\
\quad = 5.09 \ kN \quad (5.1 \ kN)
\]
A gate with width \( w \) is mounted on a frictionless hinge at the bottom of a water reservoir. Find an expression for the force, \( P \), needed to hold the gate in the position shown.

The hydrostatic pressure acting on the surface of the gate creates a downward force, equal to the weight of the water column above the gate.

The gate forms a 3–4–5 right triangle with the horizontal. Projecting the surface area of the gate into a plane which is parallel to the water surface, the total effective area of the gate is

\[
A_{\text{eff}} = w \left( \frac{5}{3} h \right)
\]

Using an average height corresponding to the half of the reservoir height, the hydrostatic force can be calculated as

\[
F = \gamma h_c A_{\text{eff}}
\]

\[
= \gamma \left( \frac{h}{2} \right) \left( \frac{5}{3} w h \right)
\]

\[
= \frac{5}{6} \gamma h^2 w
\]

The sum of the moments about the hinge is zero:

\[
\sum M_{\text{hinge}} = 0:
\]

\[
0 = P \left( \frac{5}{3} h \right) - F \left( \frac{5}{3} \right) \left( \frac{1}{3} h \right)
\]

\[
P = \frac{1}{3} F
\]

\[
= \frac{1}{3} \left( \frac{5}{3} \gamma h^2 w \right)
\]

\[
= \frac{5}{15} \gamma h^2 w
\]
Fluid Flow Measurement

Velocity (m/sec) $V$
Flow Rate (m$^3$/sec) $Q$

$Q = VA$
$V_1A_1 = V_2A_2$

A pipe carries oil at a rate of 100 liters/sec. If the internal diameter of the pipe is 10 cm, what is the velocity of flow?

Solution: $Q$ is 100 l/sec or .1m$^3$/sec. Area of the pipe is .0078 m$^2$ ($\pi r^2$) therefore

$V = Q/A = .1/.0078 = 12.8$ m/sec
Fluid Flow Measurement

Water flows at 10 m/s in a 1 cm inside diameter pipe. What is most nearly the velocity if the pipe suddenly increases in diameter to 2 cm?

\[
\begin{align*}
Q_2 &= Q_1 \\
v_2 A_2 &= v_1 A_1 \\
v_2 &= v_1 \frac{A_1}{A_2} \\
&= v_1 \frac{d_1^2}{d_2^2} \\
&= (10 \ \frac{m}{s}) \left( \frac{0.01 \ m}{0.02 \ m} \right)^2 \\
&= 2.5 \ m/s \ (3 \ m/s)
\end{align*}
\]
Fluid Flow Measurement

Water at 20°C flowing through a long, 0.10 m diameter pipe enters a section of 0.025 m diameter pipe and then flows back into a 0.10 m diameter pipe, as illustrated. All pipes are made of cast iron, and the average velocity and pressure 10 m upstream of the smaller pipe are 12 m/s and 2.0 MPa, respectively. Points A, B, and C are as labeled on the illustration. What is most nearly the fluid velocity at point B?

The flow rate, \( Q \), is constant.

\[
Q = v_A A_A = v_B A_B
\]

\[
v_B = v_A \left( \frac{A_A}{A_B} \right)
\]

\[
= v_A \left( \frac{D_A^2}{D_B^2} \right)
\]

\[
= \left( 12 \ \text{m/s} \right) \left( \frac{0.10 \ \text{m}}{0.025 \ \text{m}} \right)^2
\]

\[
= 192 \ \text{m/s} \quad (190 \ \text{m/s})
\]
Fluid Flow Measurement

Water flows through an 8 cm inside diameter pipe at a constant rate of 0.03 m³/s. The water has a kinematic viscosity of 9.609 × 10⁻⁵ m²/s.

\[ v = \frac{Q}{\left(\frac{\pi}{4}\right)D^2} = \frac{0.03 \text{ m}^3/\text{s}}{\left(\frac{\pi}{4}\right)(8 \text{ cm})(\frac{1 \text{ m}}{100 \text{ cm}})^2} \]

\[ = 5.968 \text{ m/s} \]

\[ Re = \frac{vD}{\nu} = \frac{(5.968 \text{ m/s})(0.08 \text{ m})}{9.609 \times 10^{-5} \text{ m}^2/\text{s}} \]

\[ = 4969 \quad \text{[turbulent]} \]

If a 5 cm diameter sharp-edged orifice plate is inserted in the pipe, the static pressure drop across the orifice is most nearly

\[ Q = CA \sqrt{\frac{2\Delta p}{\rho}} \]

\[ 0.03 \text{ m}^3/\text{s} = (0.61)\left(\frac{\pi}{4}\right)(5 \text{ cm})(\frac{1 \text{ m}}{100 \text{ cm}})^2 \times \sqrt{\frac{2\Delta p}{1000 \frac{\text{kg}}{\text{m}^3}}} \]

\[ \Delta p = 313.6 \times 10^2 \text{ Pa (310 kPa)} \]

(A) 200 kPa

(B) 260 kPa

(C) 310 kPa

(D) 380 kPa

From the NCEES Handbook, \( C = 0.61 \).
Fluids Bernoulli's Equation

\[ \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \]

units of \( \frac{p}{\gamma} = \frac{kN/m^2}{kN/m^3} = m \)

A pressure of 150 kPa is measured in a pipe. What is the pressure energy (pressure head)?

\[ \frac{p}{\gamma} = \frac{150}{9.8} = 15.2 \text{ m} \]
There is a 5 cm diam hole in the bottom of a 3 m tall barrel of oil (SG=.9). What is the flowrate of oil out of this hole?

Solution:

\[
Q = A \sqrt{2gh} = .00196 \sqrt{2(9.8)(3)} = .015 \text{ m}^3/\text{sec} = 15 \text{ l/sec}
\]
Fluids Bernoulli's Equation

An oil with a specific gravity of 0.92 is in a tank pressurized to 25 kPa (gage). The oil flows through a sharp-edged orifice with a diameter of 1 cm, 3 m below the oil surface.

\[ \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 \]

If the coefficient of discharge is 0.72, the discharge rate is most nearly

\[ Q = C_d A \sqrt{2g \left( h + \frac{p}{\rho g} \right)} \]
\[ = (0.72) \left( \frac{\pi}{4} \right) \left( 1 \text{ cm} \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 \times \sqrt{2 \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) \times \left( 3 \text{ m} + \frac{25 \text{ kPa}}{1000 \frac{\text{Pa}}{\text{kPa}}} \left( 0.92 \right) \left( 1000 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) \right)} \]
\[ = 6.02 \times 10^{-4} \text{ m}^3/\text{s} \quad (6 \times 10^{-4} \text{ m}^3/\text{s}) \]
Fluids Bernoulli's Equation

Water is pumped up a hill into a reservoir by a pump at the bottom of the hill. The pump discharges water at a rate of 2 m/s and a pressure of 1000 kPa. Disregarding friction, the highest possible elevation of the reservoir’s water surface is most nearly

\[
\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1
\]

\[
z_2 = h = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} = \frac{(1000 \text{ kPa})(1000 \frac{\text{Pa}}{\text{kPa}})}{9810 \frac{\text{m}^3}{\text{N}}} + \frac{(2 \frac{\text{m}}{\text{s}})^2}{9810 \frac{\text{m}^3}{\text{N}}}
\]

\[H = 102.1 \text{ m}\]

The potential energy is zero at the centerline of the pump. The pressure energy is

\[
E_p = \frac{p}{\rho} = \frac{(1000 \text{ kPa})(1000 \frac{\text{Pa}}{\text{kPa}})}{1000 \frac{\text{kg}}{\text{m}^3}} = 1000 \text{ J/kg}
\]

The velocity energy is

\[
E_v = \frac{v^2}{2} = \frac{(2 \frac{\text{m}}{\text{s}})^2}{2} = 2 \text{ J/kg}
\]

The total energy is the sum of the pressure energy and the velocity energy, as follows.

\[
E_t = E_p + E_v = 1000 \frac{\text{J}}{\text{kg}} + 2 \frac{\text{J}}{\text{kg}} = 1002 \text{ J/kg}
\]

The available energy at the surface is only potential energy.

\[
E_{t,2} = E_{t,1} = E_{p,2}
\]

\[
z_2 g = 1002 \text{ J/kg}
\]

So,

\[
\frac{E_{t,2}}{g} = \frac{1002 \frac{\text{J}}{\text{kg}}}{9.81 \frac{\text{m}}{\text{s}^2}} = 102.1 \text{ m}
\]
**Fluids Bernoulli's Equation**

A tank having a uniform cross-sectional area of 3 m$^2$ contains water at 300K with a depth of 2.2 m. The tank is open to the atmosphere. The water is drained through a 0.02 m$^2$ hole in the bottom of the tank. The time it takes to completely drain the tank is most nearly

The exit velocity depends on the elevation $z$ in the tank. Applying Bernoulli’s equation between points 1 and 2 gives

\[
\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2
\]

However, $p_1 = p_2$, $v_1 \approx 0$, and $z_2 = 0$, so

\[
z = \frac{v_2^2}{2g}
\]

\[
v_2 = \sqrt{2gz}
\]

\[
Q = A_1 \left(- \frac{dz}{dt}\right) = A_2 v_2
\]

\[
= A_2 \sqrt{2gz}
\]

Separating the variables and integrating gives

\[
\int_0^t dt = - \int_{z=2.2}^0 \left( \frac{A_1}{A_2} \right) \left( \frac{dz}{\sqrt{2gz}} \right)
\]

\[
= \left( - \frac{A_1}{A_2 \sqrt{2g}} \right) \left( \frac{\sqrt{z}}{\frac{1}{2}} \right) \bigg|_{2.2\ m}^0
\]

\[
= \left( - \frac{3\ m^2}{0.02\ m^2 \sqrt{(2)(9.81\ m/s^2)}} \right) \left( \frac{\sqrt{2.2\ m}}{\frac{1}{2}} \right)
\]

\[
= 100.5\ s \quad (100\ s)
\]
Fluids Venturi Meter

\[
\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1
\]

\[
\frac{v_2^2 - v_1^2}{2g} = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2
\]

\[
v_1 = \frac{A_2}{A_1} v_1
\]

\[
\frac{v_2^2(1 - \left(\frac{A_2}{A_1}\right)^2)}{2g} = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2
\]

\[
v_2 = \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2\right)}
\]

\[
Q = A_2 v_2 = A_2 \sqrt{2g\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2\right)}
\]
Fluids Venturi Meter

In the above venturi, if the diam of the pipe is 10cm and the throat diam is 5cm, what is the volume flow rate (liters/sec) of water through it if the pipe pressure is 300 kPa and the throat pressure is 270kPa?

Solution: \[ A_2 = .00196 \text{ m}^2 \quad A_2/A_1 = \frac{1}{4} \quad C=1 \]

\[
Q = \frac{.00196}{\sqrt{1 - .25}} \sqrt{2(9.8)(300 - 270)/9.8} = .016 \text{ m}^3/\text{sec} = 16 \text{ liters/sec}
\]
Fluids Venturi Meter

Water flows steadily through the contraction shown.

![Fluids Venturi Meter Diagram](image)

\[ D_1 = 10 \text{ cm} \quad D_2 = 5 \text{ cm} \quad h = 6 \text{ cm} \]

The velocity at section 1 is most nearly

(A) 1.0 m/s
(B) 1.4 m/s
(C) 1.8 m/s
(D) 2.2 m/s

Bernoulli's equation is

\[
\frac{p_1}{\rho_w g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_w g} + \frac{v_2^2}{2g} + z_2
\]

\[ z_1 = z_2 \]

\[ v_2 = \left( \frac{D_1}{D_2} \right)^2 v_1 = \left( \frac{10 \text{ cm}}{5 \text{ cm}} \right)^2 v_1 = 4v_1 \]

Substituting into Bernoulli's equation,

\[
v_1 = \sqrt{\left( \frac{2g}{15} \left( \frac{\rho_{\text{Hg}}}{\rho_w} - 1 \right) \right) h}
\]

\[
= \sqrt{\left( \frac{2 \left( 9.81 \frac{m}{s^2} \right)}{15} \right) (13.58 - 1)(0.06 \text{ m})}
\]

\[= 0.9936 \text{ m/s} \quad (1.0 \text{ m/s})\]
Fluids Venturi Meter

Water flows upward through a venturi meter, as shown. The differential manometer deflection is 50 cm of liquid of specific gravity 1.3.

If the coefficient of velocity is 0.95, the flow rate is most nearly?

\[ Q = C_v A_2 \sqrt{\frac{2g\left(\frac{\Delta p}{\rho_w g}\right)}{1 - \left(\frac{A_2}{A_1}\right)^2}} \]

\[ \frac{\Delta p}{\rho_w g} = \frac{(\rho_m - \rho_w)g\Delta h}{\rho_w g} = (SG_m - 1)\Delta h \]

\[ = (1.3 - 1)(50 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) \]

\[ = 0.15 \text{ m} \]

\[ = (0.95) \left( \frac{\pi}{4} \right) (0.04 \text{ m})^2 \sqrt{\frac{2(9.81 \frac{\text{m}}{\text{s}^2})(0.15 \text{ m})}{1 - \left(\frac{4 \text{ cm}}{8 \text{ cm}}\right)^4}} \]

\[ = 0.00212 \text{ m}^3/\text{s} \quad (0.0021 \text{ m}^3/\text{s}) \]
Fluids Pitot Tube

\[
\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1
\]

\[
v_1 = \sqrt{2g \left(\frac{P_2 - P_1}{\gamma}\right)}
\]
**Fluids Pitot Tube**

Water flows through a pipe with an inside diameter of 10 cm as shown.

Use the equation for fluid velocity measured by a pitot tube.

\[
v_2 = \sqrt{2g\left(\frac{\rho_{Hg}}{\rho_w} - 1\right)h}
\]

\[
= \sqrt{(2)\left(9.81 \text{ m/s}^2\right)\left(\frac{13.59}{1} - 1\right)}
\]

\[
\times (6 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)
\]

\[
= 3.85 \text{ m/s}
\]

\[
Q = Av_2
\]

\[
= \left(\frac{\pi}{4}\right)\left(10 \text{ cm}\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)\right)^2 \left(3.85 \text{ m/s}\right)
\]

\[
= 3.02 \times 10^{-2} \text{ m}^3/\text{s} \quad (0.03 \text{ m}^3/\text{s})
\]

If the deflection of the manometer is 6 cm of mercury, the flow rate is most nearly
Fluids Pitot Tube

A static pressure gauge and mercury manometer are connected to a 50.8 cm diameter pipeline flowing full of water. One cubic centimeter of mercury has a weight of 0.1336 N. What is most nearly the velocity at the center of the pipeline?

The stagnation pressure is

\[
p_0 = \left( 0.1336 \frac{N}{cm^3} \right) (25.4 \text{ cm}) \left( 100 \frac{cm}{m} \right)^2 \\
- \left( 9810 \frac{N}{m^3} \right) \left( \frac{1 m}{100 \text{ cm}} \right) (50.8 \text{ cm} + 25.4 \text{ cm})
\]
\[
= 26459 \text{ Pa}
\]

The static pressure is

\[
p_s = (60.96 \text{ cm}) \left( 1000 \frac{kg}{m^3} \right) \left( 9.81 \frac{m}{s^2} \right) \left( \frac{1 m}{100 \text{ cm}} \right) + 10342 \text{ Pa}
\]
\[
= 16322 \text{ Pa}
\]

\[
v = \sqrt{\frac{2(p_0 - p_s)}{\rho}}
\]
\[
= \sqrt{\frac{(2)(26459 \text{ Pa} - 16322 \text{ Pa})}{1000 \frac{kg}{m^3}}}
\]
\[
= 4.5 \text{ m/s}
\]
THE IMPULSE-MOMENTUM PRINCIPLE
The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

\[ \Sigma F = \Sigma Q_2 \rho_2 v_2 - \Sigma Q_1 \rho_1 v_1, \text{ where} \]

\[ \Sigma F \] = the resultant of all external forces acting on the control volume

\[ \Sigma Q_1 \rho_1 v_1 \] = the rate of momentum of the fluid flow entering the control volume in the same direction of the force

\[ \Sigma Q_2 \rho_2 v_2 \] = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force
Impulse Momentum

For the following figure, area $A$ is 0.02 m$^2$, density $\rho$ is 1000 kg/m$^3$, and velocity $v$ is 20 m/s.

The force $F$ on the stationary plate is most nearly

(A) 4000 N
(B) 6000 N
(C) 8000 N
(D) 10 000 N
Impulse Momentum

For the following figure, area $A$ is 0.02 m$^2$, density $\rho$ is 1000 kg/m$^3$, and velocity $v$ is 20 m/s.

Solution

Use the momentum equation for steady, incompressible flow.

\[ -F = \frac{\rho}{m^3} (v_{2x} - v_{1x}) \]

\[ F = -\left( 1000 \text{ kg/m}^3 \right) (0.02 \text{ m}^2) (20 \text{ m/s}) (0 - 20 \text{ m/s}) \]

\[ = 8000 \text{ N} \]

The force $F$ on the stationary plate is most nearly

(A) 4000 N
(B) 6000 N
(C) 8000 N
(D) 10000 N
Impulse Momentum

Approximately what depth of water, \( h \), will produce a horizontal force of 2.5 N against the 2 cm \( \times \) 2 cm plate?

\[
\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1
\]

\[ z_2 = h = \frac{v_1^2}{2g} \]

From the impulse-momentum theorem,

\[
F = m \Delta v = \rho v A v = v^2 A \rho
\]

\[ v_1^2 = \frac{F}{\rho A} \]

Therefore,

\[
h = \frac{v^2}{2g} = \frac{F}{2g A \rho} = \frac{F}{2g A \rho} \frac{2.5 \text{ N}}{(2)(9.81 \frac{\text{m}}{\text{s}^2})} \pi \left( \frac{0.01}{2} \text{ m} \right)^2 \left( 1000 \frac{\text{kg}}{\text{m}^3} \right)
\]

\[ = 1.62 \text{ m} \quad (1.6 \text{ m}) \]
Friction

In a 10 m section of straight, level pipe with 10 cm ID, what will be the head loss due to friction if 1 l/sec of water is flowing?

Solution: first, \( V = Q/A = (.001 \text{ m}^3/\text{sec})/0.000078 = 12.8 \text{ m/sec} \)

let’s assume \( f = .02 \) (see the Moody’s diagram), then

\[
h_f = .02 \times \frac{10}{.01} \left\{ \frac{12.8^2}{2(9.8)} \right\} = 167 \text{ m}
\]
Drag

A large spherical balloon, 10 m in diameter, is to be propelled through 10° C air at 10 m/s. The kinematic viscosity and the density of the air are $1.39 \times 10^{-5}$ m²/s and 1.2 kg/m³, respectively. The power needed to propel the balloon is most nearly

First find the Reynolds number for the flow, $v$ is the kinematic viscosity, $v$ is the velocity and $D$ is the diameter of flow.

$$Re = \frac{vD}{\nu} = \frac{(10 \text{ m})(10 \text{ m})}{(1.39 \times 10^{-5} \text{ m}^2 / \text{s})} = 7 \times 10^6$$

Use Re to find the drag coefficient for a sphere using the drag coefficient versus Re chart.

$$C_D = 0.15$$

Find the drag force over a sphere. $A$ is the cross-sectional area and $\rho$ is the density of air.

$$F_D = \frac{C_D A \rho v^2}{2} = \frac{(0.15)\pi(5 \text{ m})^2 \left(1.2 \frac{kN}{m^3}\right)(10 \text{ m})^2}{2} = 707 \text{ N}$$

Finally, multiply the force by the velocity to find the power.

$$\dot{w} = F_D v = (707 \text{ N})(10 \text{ m/s}) = (7070 \text{ W})(\frac{1 \text{ Hp}}{746 \text{ W}}) = 9.5 \text{ Hp}(10 \text{ Hp})$$